

Density dependence of nuclear symmetry energy constrained by mean-field calculations

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We establish a correlation for the symmetry energy at saturation density S_0 , slope parameter L and curvature parameter K_{sym} based on widely different mean field interactions. With the help of this correlation and available empirical and theoretical information, the density dependent behavior around the saturation density is determined. We compare the results obtained with the present approach with those by other analyses. With this obtained density dependent behavior of the symmetry energy, the neutron skin thickness of ^{208}Pb and some properties of neutron stars are investigated. In addition, it is found that the expression $S(\rho) = S_0(\rho/\rho_0)^\gamma$ or $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ does not reproduce the density dependence of the symmetry energy as predicted by the mean-field approach around nuclear saturation density.

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I. INTRODUCTION

Our knowledge about the features of nuclear matter at saturation density ρ_0 is based primarily on the masses of nuclei, like the density $\rho_0 = 0.16 \text{ fm}^{-3}$, energy per particle $a_v = -16 \text{ MeV}$ and symmetry energy $S(\rho = \rho_0) = 28 - 34 \text{ MeV}$ [1]. However, the variation of the symmetry energy with density is still intensely debated [2–5]. The symmetry energy which characterizes the isospin-dependent part of the equation of state (EOS) of asymmetric nuclear matter, plays a crucial role in many issues of nuclear physics as well as astrophysics. It relates the heavy ion reactions [6–13], stability of superheavy nuclei [14], fusion cross sections [15] and structures, composition and cooling of neutron stars [16–19]. Many theoretical and experimental efforts have been performed to constrain the density-dependent symmetry energy [3–8].

The energy per particle of nuclear matter with density $\rho = \rho_n + \rho_p$ and asymmetry $\delta = (\rho_n - \rho_p)/\rho$ is usually written as $e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2$, where ρ_n , ρ_p and ρ are the neutron, proton and nucleon densities. Around the nuclear matter saturation density ρ_0 , the symmetry energy $S(\rho)$ can be expanded to second order in density as

$$S(\rho) = S_0 + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 + \mathcal{O} \left[\left(\frac{\rho - \rho_0}{\rho_0} \right)^3 \right], \quad (1)$$

where $L = 3\rho \partial S(\rho)/\partial \rho|_{\rho_0}$ and $K_{\text{sym}} = 9\rho^2 \partial^2 S/\partial \rho^2|_{\rho_0}$ are the slope and curvature parameters at ρ_0 that govern the density dependence of $S(\rho)$ around ρ_0 . S_0 describes the symmetry energy at density ρ_0 . Recently, some progress has been made in determining the density dependence of $S(\rho)$ around the saturation density ρ_0 from nuclear isospin diffusion, double n/p ratio in intermediate energy heavy-ion collisions, pygmy dipole resonance, neutron skin thickness and the nuclear binding energy. A description of isospin diffusion data with a symmetry energy of $S(\rho) = S_0(\rho/\rho_0)^\gamma$ with $\gamma = 0.69 - 1.05$ has been obtained by using an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model [20], and a value of $\gamma = 0.5$ is inferred from the preequilibrium neutron and proton transverse emissions comparing to IBUU transport calculations [21]. The giant dipole resonance (GDR) of ^{208}Pb analyzed with the Skyrme interactions implies $\gamma = 0.5 - 0.65$ [22]. The $S(\rho)$ extracted from more than 2000 measured nuclear masses gives $\gamma = 0.6 - 0.8$ [23]. More recently, the analysis of isospin diffusion and double ratio data involving neutron and proton spectra by an improved quantum molecular dynamics transport model suggests $\gamma = 0.4 - 1.05$ with $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ [24]. Although significant progress was made in determining the symmetry energy, it remains an open question nowadays.

It has been established that the neutron skin thickness ΔR_{np} , given by the difference of neutron and proton root-mean-square radii of heavy nuclei, correlates linearly with the slope L around the saturation [25–30]. Although the theoretical predictions on $S(\rho)$ with the current nuclear mean field methods and neutron skin thickness are extremely diverse, this correlation is universal in the realm of mean field theory as it is based on widely different nu-

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clear functionals [29, 30]. Based on the similar idea, in this work, we try to extract a relation of the three quantities S_0 , L and K_{sym} in widely different mean field interactions to constrain the density dependent symmetry energy $S(\rho)$. This work is organized as follows. In Sec. II, we extract a relation between S_0 , L and K_{sym} universally within the mean field framework since it is based on widely different nuclear mean-field interactions. In sec. III, by employing this relation and other considerations, we determine the density dependence of the symmetry energy around the saturation density. With the obtained density dependent behavior of the symmetry energy, the neutron skin thickness of ^{208}Pb and some properties of neutron stars are investigated. Finally, a short summary is given in Sec. IV.

II. ESTABLISHMENT OF THE RELATION FOR THREE QUANTITIES S_0 , L AND K_{sym}

Let us first establish a relation of the three quantities. With the relation $S(\rho) = S_0(\rho/\rho_0)^\gamma$ describing the density dependence of the symmetry energy [20], one can obtain $L = 3S_0\gamma$ and $K_{\text{sym}} = 9S_0\gamma(\gamma - 1)$, and thus a correlation of S_0 , L and K_{sym} can be derived

$$S_0 = \frac{L}{3 + K_{\text{sym}}/L}. \quad (2)$$

For the other density dependent behavior $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$, we have $L = 25 + 3(S_0 - 12.5)\gamma$ and $K_{\text{sym}} = -25 + 9C_p\gamma(\gamma - 1)$. Then, the correlation of S_0 , L and K_{sym} takes the form

$$S_0 = 12.5 + \frac{(L - 25)^2}{3L + K_{\text{sym}} - 50}. \quad (3)$$

The shape of the density dependence of the symmetry energy $S(\rho)$ that from the density-dependent M3Y (DDM3Y) interaction [32] can be written as $S(\rho) = C_k(\rho/\rho_0)^{2/3} + C_1(\rho/\rho_0) + C_2(\rho/\rho_0)^{5/3}$ with $C_k = (2^{2/3} - 1)\frac{5}{3}\frac{\hbar^2 k_F^2}{2m} = 13.0$ MeV. S_0 , L and K_{sym} can be expressed by parameters C_1 and C_2 with $S_0 = 13.0 + C_1 + C_2$, $L = 26.0 + 3C_1 + 5C_2$ and $K_{\text{sym}} = -26 + 10C_2$. Therefore, the correlation is given by

$$S_0 = 2.6 + \frac{L}{3} - \frac{K_{\text{sym}}}{15}. \quad (4)$$

We now test whether Eqs. (2), (3) and (4) work well or not by using widely different mean field functionals including relativistic and non-relativistic versions. As done in Ref. [31], to prevent eventual biases in our study, we avoid including more than two models of the same kind fitted by the same group. We also avoid models yielding a charge radius of ^{208}Pb away from experiment data by more than 1% as in Ref. [31] since we will study the neutron skin thickness of ^{208}Pb . The interactions we used here are (1)LNS1, (2)LNS5, (3)MSL0,

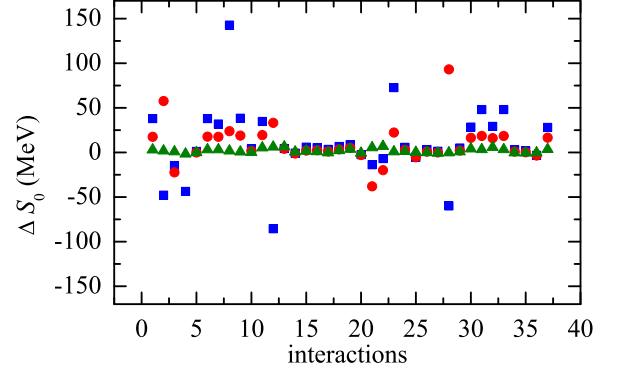


FIG. 1: (Color online) $\Delta S_0 = S_0 - L/(3 + K_{\text{sym}}/L)$ (rectangle symbols), $\Delta S_0 = S_0 - 12.5 - (L - 25)^2/(3L + K_{\text{sym}} - 50)$ (circle symbols) and $\Delta S_0 = S_0 - 2.6 - L/3 - K_{\text{sym}}/15$ (triangle symbols) within the density dependent behavior $S(\rho) = S_0(\rho/\rho_0)^\gamma$, $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ and DDM3Y shape, respectively. The horizontal ordinate denotes the sequence number for the interactions mentioned in text.

(4)SIV, (5)SkT4, (6)T6, (7)SkP, (8)SkM*, (9)SkX, (10)PK1, (11)D1S, (12)SLy4, (13)FSUGold, (14)SkMP, (15)SkI5, (16)NLSH, (17)TM1, (18)NL3, (19)NL1, (20)Sk255, (21)DDME1, (22)DDME2, (23)DDM3Y, (24)PC-F1, (25)Ska, (26)SV, (27)QMC, (28)MSkA, (29)SkI2, (30)MSk7, (31)HFB-17, (32)BSk8, (33)BSk17, (34)GM1, (35)GM3, (36)Sk272, (37)v090. The root-mean-square deviations given by Eqs. (2), (3) and (4) are 38.65 MeV, 65.29 MeV and 2.94 MeV for S_0 value, respectively. In order to give a clearer show, we define a quantity ΔS_0 to describe the differences between the left hand side and the right hand side of Eqs. (2), (3) and (4), and plot them in Fig. 1. As can be seen, ΔS_0 with Eq. (4) approximates zero for these widely different interactions while ΔS_0 with Eqs. (2), (3) tends to deviate from zero considerably. Therefore, Eqs. (2) and (3) can not be taken as accurate expressions for the description of the correlation of S_0 , L and K_{sym} displayed by the mean-field calculations. It is noted that the minimum value of K_{sym} is $-9S_0/4 \approx -72$ MeV with $S(\rho) = S_0(\rho/\rho_0)^\gamma$ and $-25 - 9C_p/4 \approx -69$ MeV with $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$. Many interactions, however, provide $K_{\text{sym}} < -100$ MeV (as will be seen in Fig. 2), which leads to large discrepancies of Eqs. (2) and (3) when compared to the mean-field predictions. This fact also indicates $S(\rho) = S_0(\rho/\rho_0)^\gamma$ or $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ is not suitable to describe the density dependent behavior of the symmetry energy around ρ_0 as predicted by the mean field approach. In particular, they cannot describe a very soft symmetry energy due to their monotonous increase with

the density. However, Eq. (4) is much better to be taken as a relationship for the correlation of S_0 , L and K_{sym} . Noting that the $S(\rho)$ with the DDM3Y shape is still approximate, Eq. (4) can be further improved to obtain the least deviation. Replacing the index $5/3$ in the DDM3Y shape by a coefficient γ , one has $S_0 = a + L/3 + bK_{\text{sym}}$. By performing a least-squares fit with the calculated S_0 , L and K_{sym} using the interactions above, the values of the parameters are $a = 3.9199$ and $b = -0.07323$ with a rms deviation of 2.12 MeV for S_0 value which is slightly different from that within the DDM3Y shape, and hence the relation is give by

$$L = -11.76 + 3S_0 + \frac{K_{\text{sym}}}{4.55}, \quad (5)$$

with a rms deviation of 6.35 MeV for L value. This formula can be considered as a universal one within the mean-field framework since it is based on widely different nuclear mean-field interactions. As a consequence, the analytical and simple expression reported in Eq. (5) describes in very good approximation the high correlations displayed between S_0 , L and K_{sym} arising from the predictions of the representative set of employed mean-field models. The corresponding expression for the density dependence of the symmetry energy is given by

$$S(\rho) = C_k \left(\frac{\rho}{\rho_0} \right)^{2/3} + C_1 \left(\frac{\rho}{\rho_0} \right) + C_2 \left(\frac{\rho}{\rho_0} \right)^{1.52}, \quad (6)$$

where $C_k = 17.47$ is larger than that in the DDM3Y shape, which perhaps can be regarded as the mass in the kinetic energy being replaced by the effective mass m^* . In this step, parameters C_1 and C_2 remain unknown.

III. DENSITY DEPENDENCE OF THE SYMMETRY ENERGY AROUND ρ_0 WITHIN THE OBTAINED RELATIONS

TABLE I: The S_0 values obtained from various independent studies in other references.

Reference	S_0 (MeV)	Reference	S_0 (MeV)
Ref. [3, 20]	31.6	Ref. [23]	31.1 ± 1.7
Ref. [24]	30.1	Ref. [33]	32.4 ± 1.1
Ref. [34]	32.0 ± 1.8	Ref. [35]	32.0
Ref. [36]	30.048 ± 0.004	Ref. [4]	32.3 ± 1.3

Of the three quantities S_0 , L and K_{sym} , S_0 value is relatively well-known. Table I lists the recent S_0 value coming from recent various studies. The largest range of $S_0 = 31.6 \pm 2.2$ MeV from Table I will be used in the present study. Yet, we have to call for an additional condition to constrain the detailed L and K_{sym} values. Recently, Centelles *et al.* found that the symmetry energy (coefficient) $a_{\text{sym}}(A)$ of a finite nucleus with mass number A is approximately equal to the symmetry energy $S(\rho_A)$ of nuclear matter at a reference density ρ_A ,

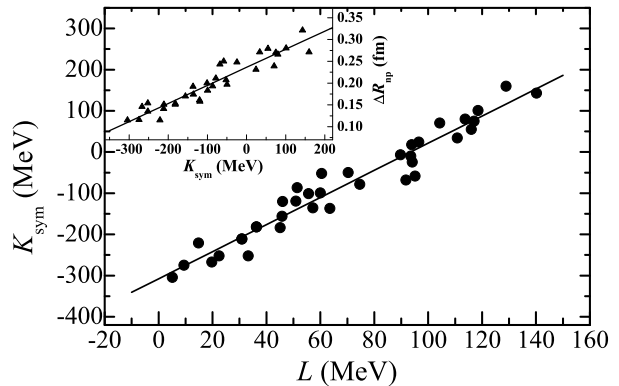


FIG. 2: Correlation of the curvature K_{sym} with the slope L of the symmetry energy at ρ_0 . The line gives the fitting result with the correlation coefficient $r = 0.972$. The inset displays the correlation between the neutron skin thickness Δr_{np} in ^{208}Pb and K_{sym} value. The line gives the fitting results with $r = 0.945$.

namely, $S(\rho_A) = a_{\text{sym}}(A)$ [29]. For a given nucleus, the ρ_A is determined, such as $\rho_A = 0.1 \text{ fm}^{-3}$ for ^{208}Pb . Here, this relationship is employed to investigate the correlation between K_{sym} and L . $a_{\text{sym}}(A)$ of a finite nucleus is given by

$$a_{\text{sym}}(A) = \frac{S_0}{1 + \kappa A^{-1/3}}, \text{ with } \kappa = \frac{9}{4} \frac{S_0}{Q}, \quad (7)$$

where Q is the surface stiffness that measures the resistance of the nucleus against separation of neutrons from protons to form a neutron skin. In Ref. [30], it is shown that S_0/Q ratio displays a linear relationship with L . Then, parameter κ can be written as $\kappa = mL + n$. By combining Eq. (7) and the expression of the symmetry energy obtained with the DDM3Y shape aforesaid, the relationship of $S(\rho_A) = a_{\text{sym}}(A)$ can be converted into $L^2 + a_1 L K_{\text{sym}} + a_2 K_{\text{sym}} + a_3 L + a_4 = 0$ with new coefficients a_1, a_2, a_3 and a_4 by replacing S_0 , C_1 and C_2 by L and K_{sym} . We fit the results from the nuclear mean field calculations with the interactions mentioned above. However, in the fitting process, it is found that the first two terms can be neglected in the region under consideration. No matter whether or not the first two terms are taken into account, one can obtain the same root-mean-square deviation $\sqrt{\langle \sigma^2 \rangle} = 29.8$ MeV for K_{sym} value. Accordingly, this correlation can be further simplified as a linear relation, as visibly shown in Fig. 2. By performing a two parameter fitting to this relationship, we have

$$K_{\text{sym}} = -307.862 + 3.292L, \quad (8)$$

with $\sqrt{\langle \sigma^2 \rangle} = 29.8$ MeV for K_{sym} value and the correlation coefficient $r = 0.972$. The results of fitting are

presented in Fig. 2 by the line. One readily sees that the predicted K_{sym} with the mean field approaches varies largely from -300 MeV to 150 MeV. By combining Eq. (5) and Eq. (8) together with $S_0 = 31.6 \pm 2.2$ MeV, in the present work, some information on the density dependence of $S(\rho)$ is investigated.

Fig. 3 illustrates the present estimated L values compared with those from other independent approaches. One can find that the present finding has a remarkable overlap with but on the whole slightly softer than the very recent results of Carbone *et al.* [4], Liu *et al.* [23] and Tsang *et al.* [24]. This, to a large extent, perhaps stems from the relation of Eq. (5) based on the formula of $S(\rho)$ that can characterize the very soft symmetry energy in a much more reasonable manner. In the other analysis with $S(\rho) = S_0(\rho/\rho_0)^\gamma$ or $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$, the slope parameter of the symmetry energy at saturation density should be overestimated. To give a clearer explanation, we present an example here to show the overestimation of the L values with $S(\rho) = S_0(\rho/\rho_0)^\gamma$. In Ref. [33], Danielewicz and Lee extracted the mass dependent symmetry energy coefficients of finite nuclei $a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1}$ with $S_0 = 27.39$ MeV and $\kappa = 1.28$ [37]. With the relation $a_{\text{sym}}(A) = S(\rho_A)$ proposed in Ref. [29], if the density dependent behavior $S(\rho) = S_0(\rho/\rho_0)^\gamma$ is adopted, one obtains $L = 34$ MeV. However, if the DDM3Y shape or the modified one (Eq. (6)) is applied, we obtain $L = 21$ MeV and $L = 24$ MeV, respectively, lower than that stemming from $S(\rho) = S_0(\rho/\rho_0)^\gamma$. With $S_0 = 29.4 - 33.8$ MeV in Table I, the L and K_{sym} values obtained with the present method are 56 ± 24 MeV and -125 ± 79 MeV, respectively. It is interesting to see that the window reported in reference of Warda *et al.* [30] for the analysis of anti-protonic atoms data and that in the present manuscript are exactly the same, as shown in Fig. 3. The S_0 value with less uncertainty will lead to narrower windows of L and K_{sym} values within our approach. The errors for L and K_{sym} arising from the errors of the parameters in the fitting are less important compared with these resulting from the uncertainty of S_0 value. Therefore, for simplicity and clarity, we only selected the optimal fitting results. The small deviation and large correlation coefficient indicate that the parameters are well constrained by the information used in the fitting procedure.

It is well known that L shows a linear relationship with neutron skin thickness ΔR_{np} of finite nuclei. In Ref. [31], this linear correlation is given by $\Delta R_{np} = 0.101 + 0.00147L$ for ^{208}Pb . Because of Eq. (8), K_{sym} should also show a linear correlation with the neutron skin thickness ΔR_{np} . Using the aforesaid interactions for finite nuclei, we obtain the correlation between ΔR_{np} in ^{208}Pb and K_{sym} by performing a fitting procedure

$$\Delta R_{np} = (0.236210 \pm 0.003991) + (0.000415 \pm 0.000026)K_{\text{sym}}, \quad (9)$$

where ΔR_{np} and K_{sym} are in units of fm and MeV, re-

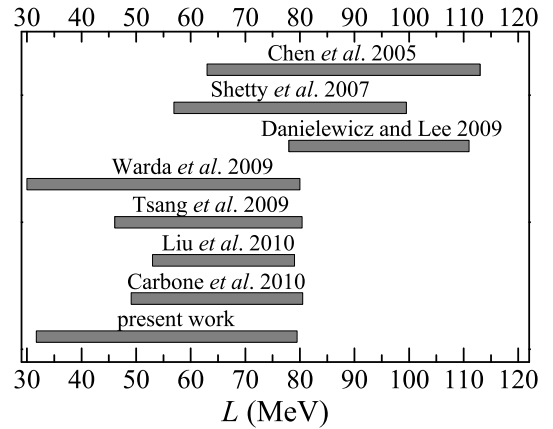


FIG. 3: Comparison between the L values obtained in the present work and those from other recently various analyses including Carbone *et al.* [4], Liu *et al.* [23], Tsang *et al.* [24], Warda *et al.* [30], Danielewicz and Lee [33], Shetty *et al.* [3] and Chen *et al.* [20].

spectively. The result of the fitting is presented in the inset of Fig. 2 by the line. A larger K_{sym} value implies a thicker neutron skin. The linear relation allows one to extract K_{sym} with the measured ΔR_{np} . Therefore, once an accurate measurement of ΔR_{np} is achieved, L as well as K_{sym} values can be determined simultaneously. As a consequence, a richer information about the density dependent behavior of the symmetry energy can be achieved. Here K_{sym} is used in turn to determine the ΔR_{np} . With the K_{sym} values obtained above with $S_0 = 31.6 \pm 2.2$ MeV, the ΔR_{np} in ^{208}Pb is estimated to be 0.185 ± 0.035 fm. With the relationship between ΔR_{np} and L in Ref. [31] together with the L values of 56 ± 24 MeV that we obtained above, the ΔR_{np} for ^{208}Pb is 0.183 ± 0.035 fm, which is quite consistent with that from $\Delta R_{np} - K_{\text{sym}}$ correlation. This fact indicates the justification of our approach to a large extent. Again, our calculated ΔR_{np} for ^{208}Pb is in good agreement with the value from analysis of PDR [4, 34]. The neutron-rich skin of a heavy nucleus is related to the properties of neutron star crusts. For instance, the thicker the neutron skin is, the thinner the solid crust of a neutron star [16]. Horowitz and Piekarewicz proposed that if ΔR_{np} for ^{208}Pb is greater than about 0.24 fm, the electron fraction becomes large enough to allow the direct URCA process to cool down a $1.4M_\odot$ neutron star [16]. Our calculated ΔR_{np} is too small to allow the direct URCA process in this canonical neutron star to occur. An almost linear relationship between the ΔR_{np} and the critical density ρ_c of a phase transition from nonuniform to uniform neutron rich matter is put forward in Ref. [16] with an approximate relation of $\rho_c \approx 0.16 - 0.39\Delta R_{np}$ (^{208}Pb).

Using this relation combining with ΔR_{np} in ^{208}Pb deduced above, one obtains $\rho_c \approx 0.09 \text{ fm}^{-3}$ which is in accord with $\rho_c \approx 0.096 \text{ fm}^{-3}$ of the microscopic EOS of Friedman and Pandharipande [38]. These two examples display the applications of the correlations we employed in understanding the physics of compact objects.

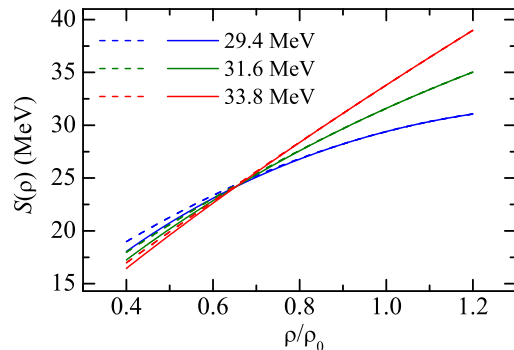


FIG. 4: (Color online) Behavior of the density dependent symmetry energy with different S_0 values from Eqs. (1) (dash lines) and (6) (solid lines), respectively.

When S_0 value is fixed, the parameters C_1 and C_2 in Eq. (6) are accordingly determined, and hence the density dependence of the symmetry energy in the form of a modified DDM3Y shape is obtained. Within this density dependent behavior of $S(\rho)$, one may investigate the symmetry energy at subsaturation densities. Fig. 4 presents the symmetry energy $S(\rho)$ as a function of the density with several detailed S_0 values. The solid (dash) curves denote the calculations with Eq. (6) (Eq. (1)). As can be seen, $S(\rho)$ shows different behaviors with different S_0 values. The symmetry energy tends to become stiffer with a larger S_0 value. When S_0 is selected to be 33.8 MeV, the relation of $S(\rho) - \rho$ is almost linear. This explains the fact that the relativistic mean field models have traditionally predicted a stiff symmetry energy—these interactions give larger S_0 values. The density dependent behavior stemming from Eq. (6) and Taylor expansion of Eq. (1) mostly coincide with each other, which indicates that the Taylor expansion of Eq. (1) can be applied in a wide density range when Eq. (6) is employed for the description of $S(\rho)$. In addition, consistent with the mean-field results of Refs. [25, 27, 29], it is found that different curves give nearly the same value for the symmetry energy at a density around $\rho = 0.1 \text{ fm}^{-3}$. The obtained symmetry energy $S(\rho) = 23.6 \pm 0.02 \text{ MeV}$ at a nucleon density of $\rho = 0.1 \text{ fm}^{-3}$ agrees with $23.3 - 24.9 \text{ MeV}$ by an analysis of the GDR of ^{208}Pb with the Skyrme interactions [22] but is larger than the values of $21.2 - 22.5 \text{ MeV}$ obtained with analysis of the GDR of ^{132}Sn within relativistic mean field models [39]. Moreover, compared with that from the PDR, our result includes a much less

uncertainty. As suggested by Daoutidis and Goriely [40], the PDR strength measurements nowadays cannot yield quantitative insight on the symmetry energy, and thus both theoretical approaches as well as phenomenological interactions need to be further improved to investigate the symmetry energy. Some authors even proposed that the PDR strength is weakly correlated with the neutron skin thickness of heavy nuclei [41] and hence the density dependence of the symmetry energy. Finally, we would note that compared with the transport models and the PDR measurements, our approach is much more straightforward. Yet, our approach is based only on mean field models and could only gain some limited information about the symmetry energy. Opposite to our method, the analysis of the experimental data within microscopic techniques, such as the random phase approximation for the study of the PDR, may allow one to investigate the internal structure and dynamics of the nucleus, and hence one can obtain more detailed knowledge. These different approaches can validate or complement each other to get more compelling results.

IV. SUMMARY

Based on the similar idea in Ref. [25–30] that the neutron skin thickness correlates with the slope parameter L around the saturation density in mean field models, we have established a relation for three quantities S_0 , L and K_{sym} in widely different mean field interactions. With this relation and other constraint conditions, the density dependence of the nuclear symmetry energy $S(\rho)$ has been investigated in the present work. With the obtained density dependence of the symmetry energy, the neutron skin thickness of ^{208}Pb and some properties of neutron stars were analyzed. The main conclusions are as follows. (i) It is not suitable to take the form of $S(\rho) = S_0(\rho/\rho_0)^\gamma$ or $S(\rho) = 12.5(\rho/\rho_0)^{2/3} + C_p(\rho/\rho_0)^\gamma$ to describe the behavior of the symmetry energy as predicted by the mean-field approach around the nuclear saturation density while the shape from the DDM3Y is much better. (ii) Based on the latter formula, we have provided an analytical and simple expression for the high correlation shown by S_0 , L and K_{sym} within the mean field framework. Therefore, if the symmetry energy at saturation density ρ_0 is known, the values of K_{sym} and L can be unambiguously related by using Eq. (5). The L and K_{sym} values in the present study are $56 \pm 24 \text{ MeV}$ and $-125 \pm 79 \text{ MeV}$ respectively with $S_0 = 31.6 \pm 2.2 \text{ MeV}$ yielded in other references. (iii) The neutron skin thickness of ^{208}Pb displays a linear correlation with K_{sym} . Thus, once the neutron skin thickness is measured accurately, not only slope parameter L but also curvature parameter K_{sym} of the symmetry energy around the saturation can be determined. Thus, a richer information about the density dependent behavior of the symmetry energy can be achieved. ΔR_{np} of ^{208}Pb is $0.185 \pm 0.035 \text{ fm}$ with the K_{sym} value in item (ii) which is too small

to allow the direct URCA process in $1.4M_{\odot}$ neutron star to occur.(iv) Within our approach, the behavior of $S(\rho)$ against ρ around the normal density almost relies on the symmetry energy at saturation density ρ_0 . We show that a large S_0 value leads to a stiff symmetry energy. They, however, naturally provide almost the same result of about 23.6 MeV at $\rho = 0.1 \text{ fm}^{-3}$.

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